

Why do we have Measurement Uncertainty?

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Measurement Uncertainty

Content

1. What is a Measurement
2. Random and Systematic Effects
3. Blunders and Spurious Errors
4. Major Goals of INC-1
5. Statistical Evaluation of Uncertainty
6. No Statistical Evaluation of Uncertainty
7. Calculation of the Combined Standard Uncertainty

Measurement

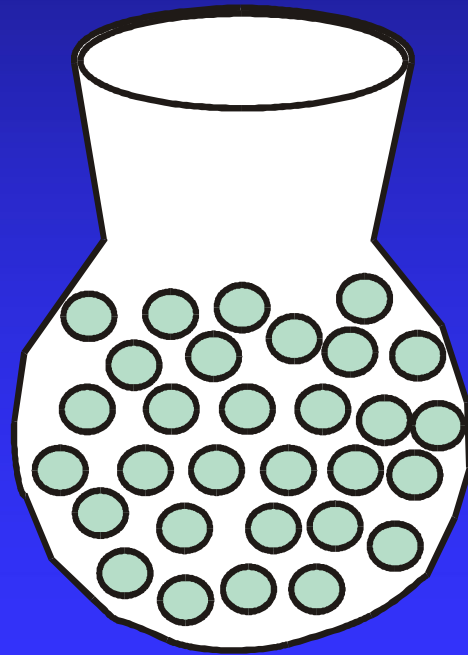


- Objective of a measurement is to determine the value of the measurand
- Appropriate specification of the measurand and the measurement procedure
- The result of a measurement is only an **estimate** of the value of the measurand

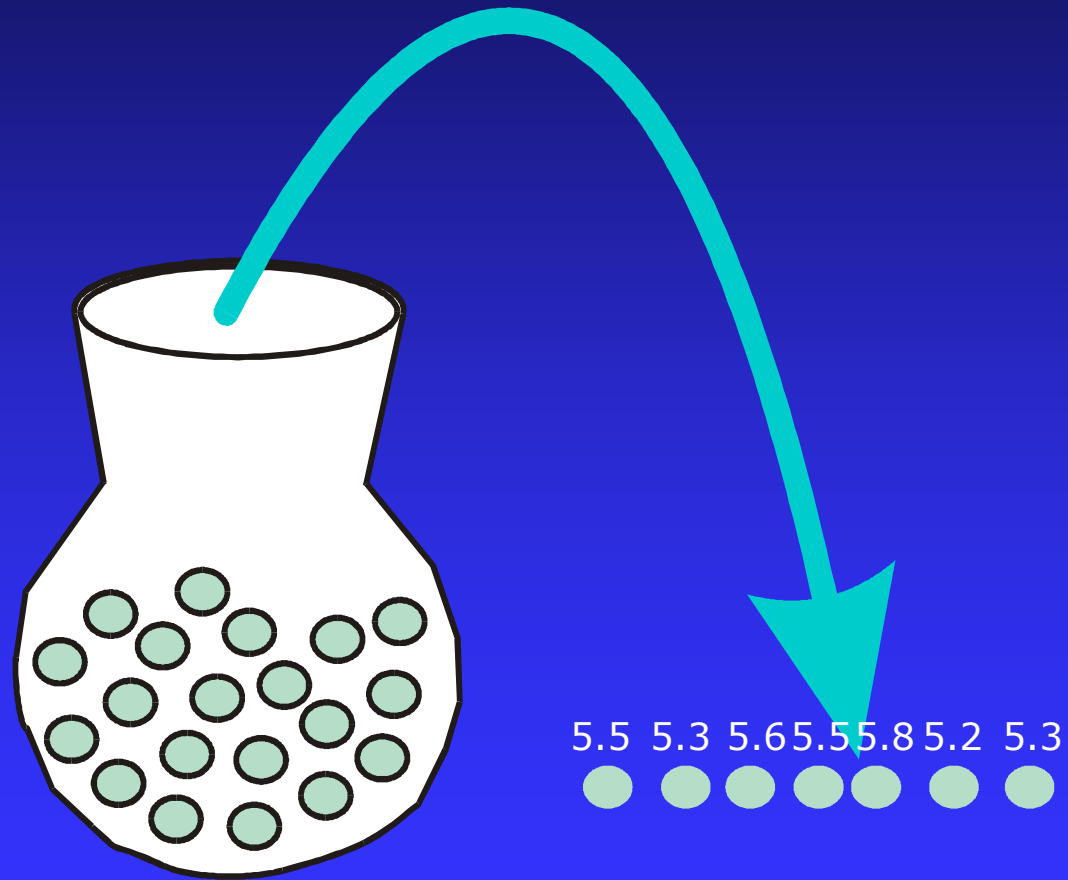
Measurement



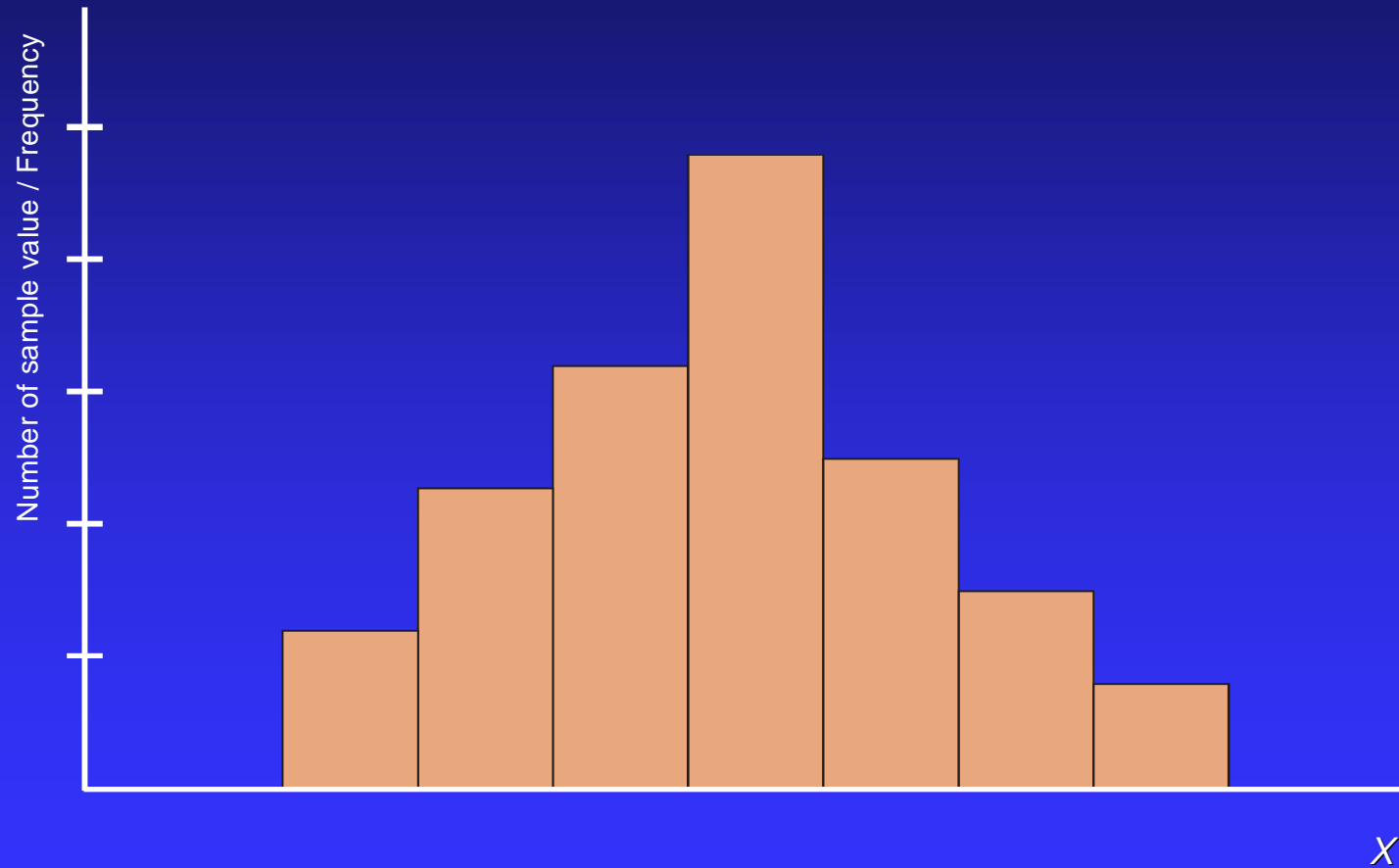
Model of the Measurement Process



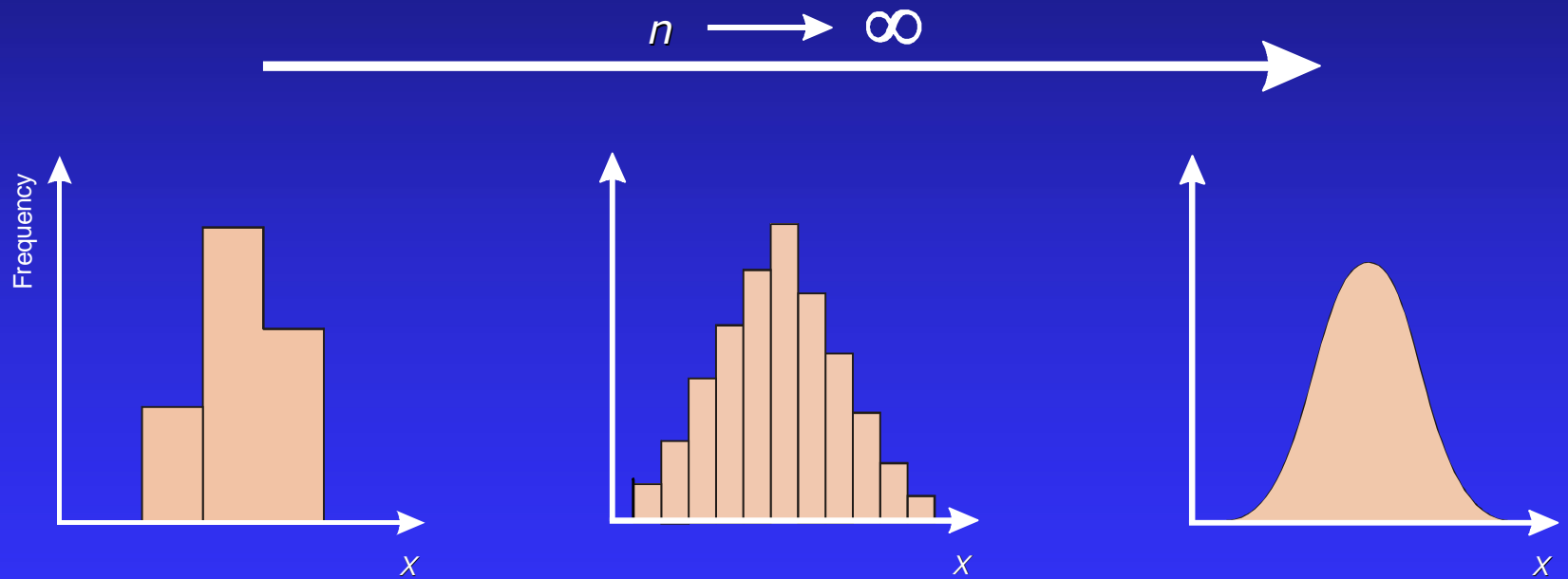
Model of the Measurement Process



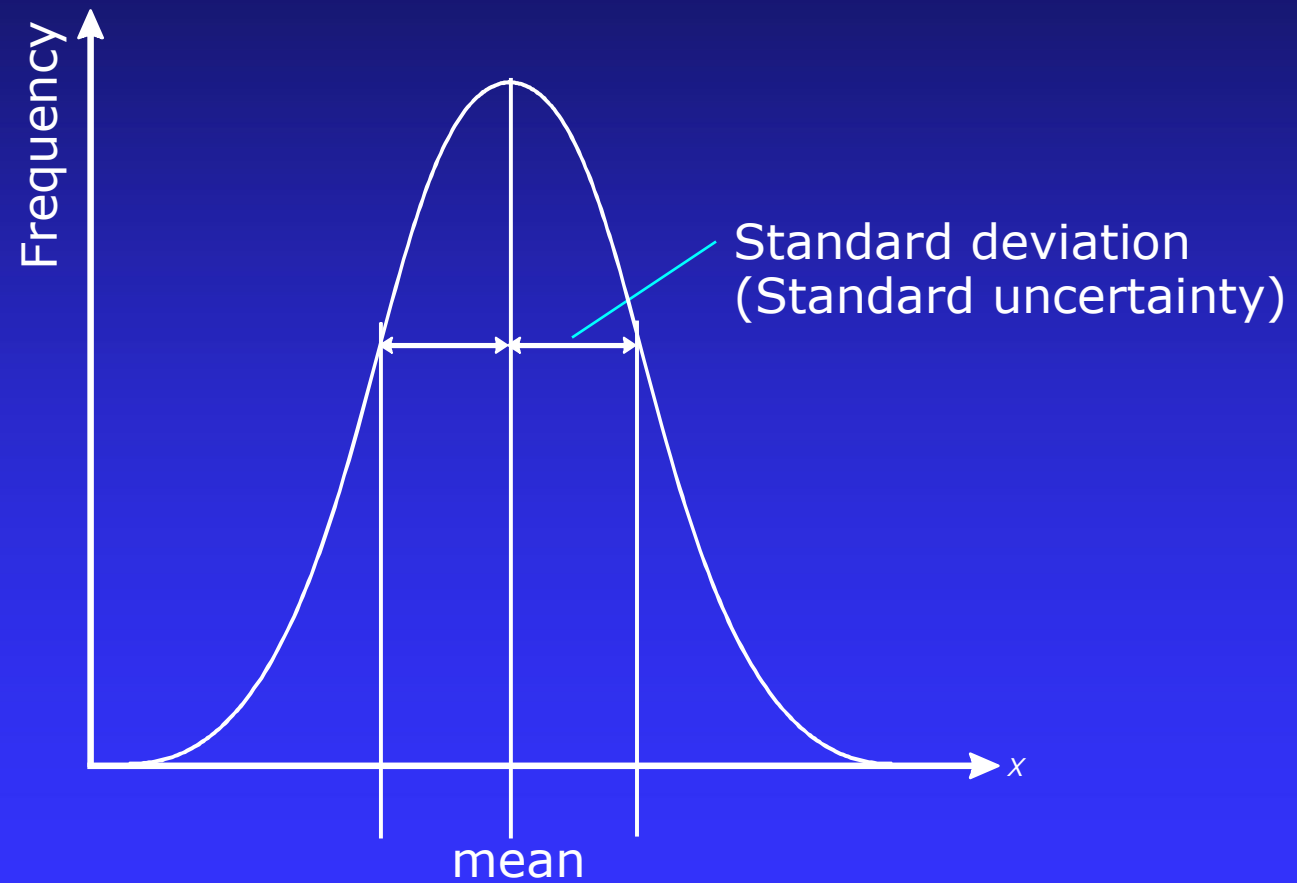
Random Effects



Random Effects



Random Effects



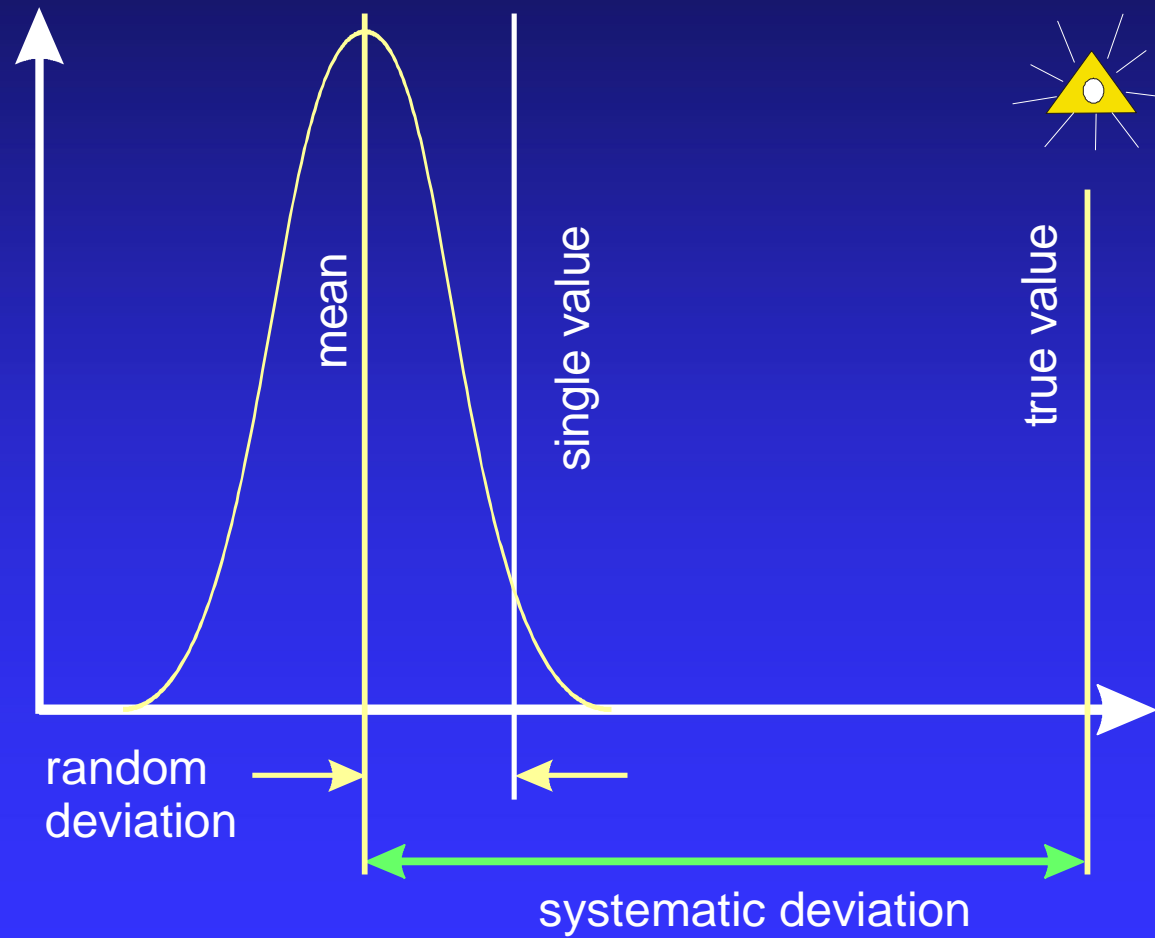
Random Effects

■ Mean:
$$\bar{x} = \frac{x_1 + x_2 + \dots + x_i + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$$

■ Standard deviation:

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

Systematic Effects



Blunders or “Spurious Errors”

8.6557 ml

6.8588 ml ⇒ correct: 8.6588 ml

8.6558 ml

8.6409 ml

8.6323 ml

8.6555 ml

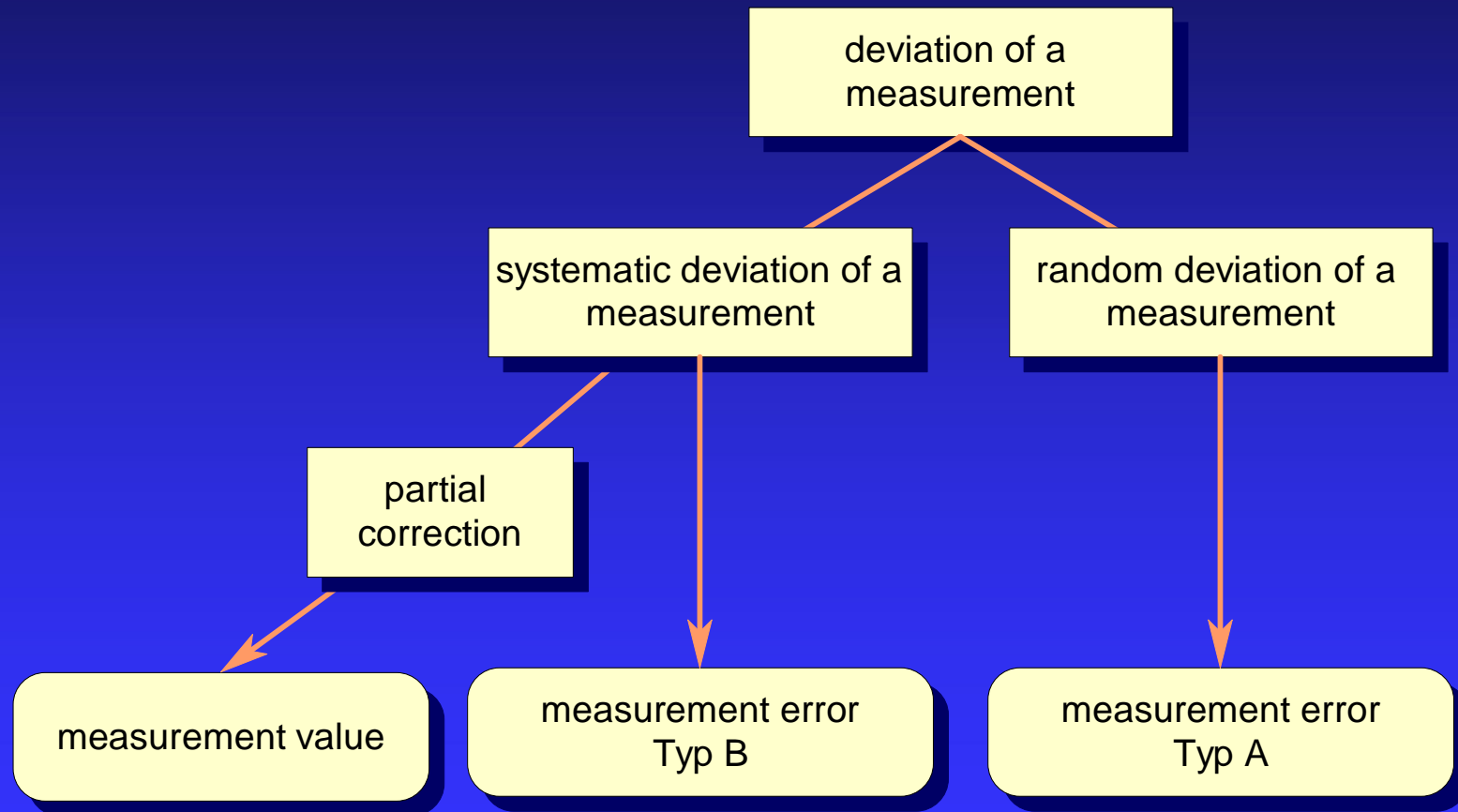
8.6731 ml

8.6367 ml ⇒ correct: 8.6637 ml

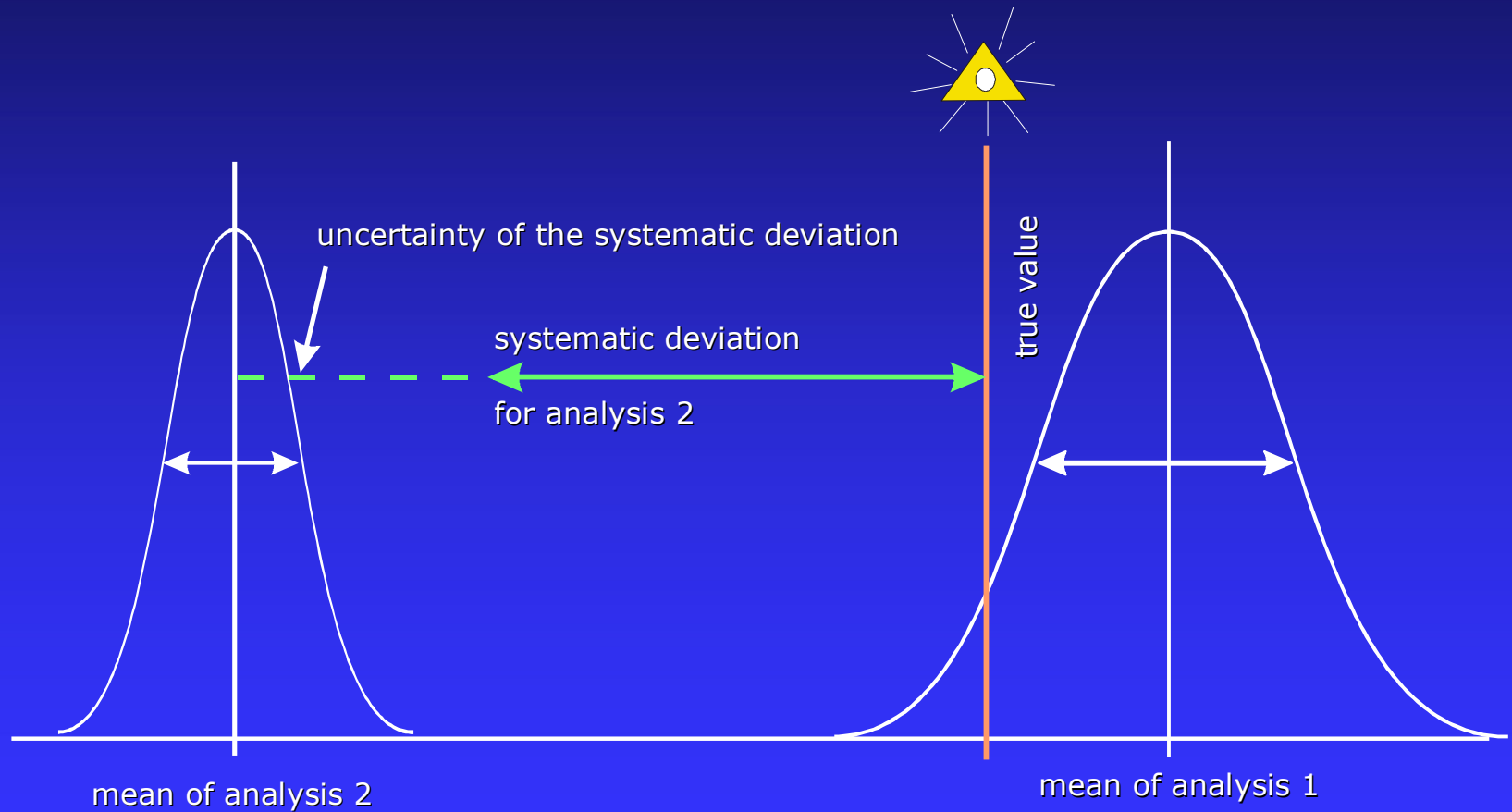
8.6577 ml

8.6746 ml

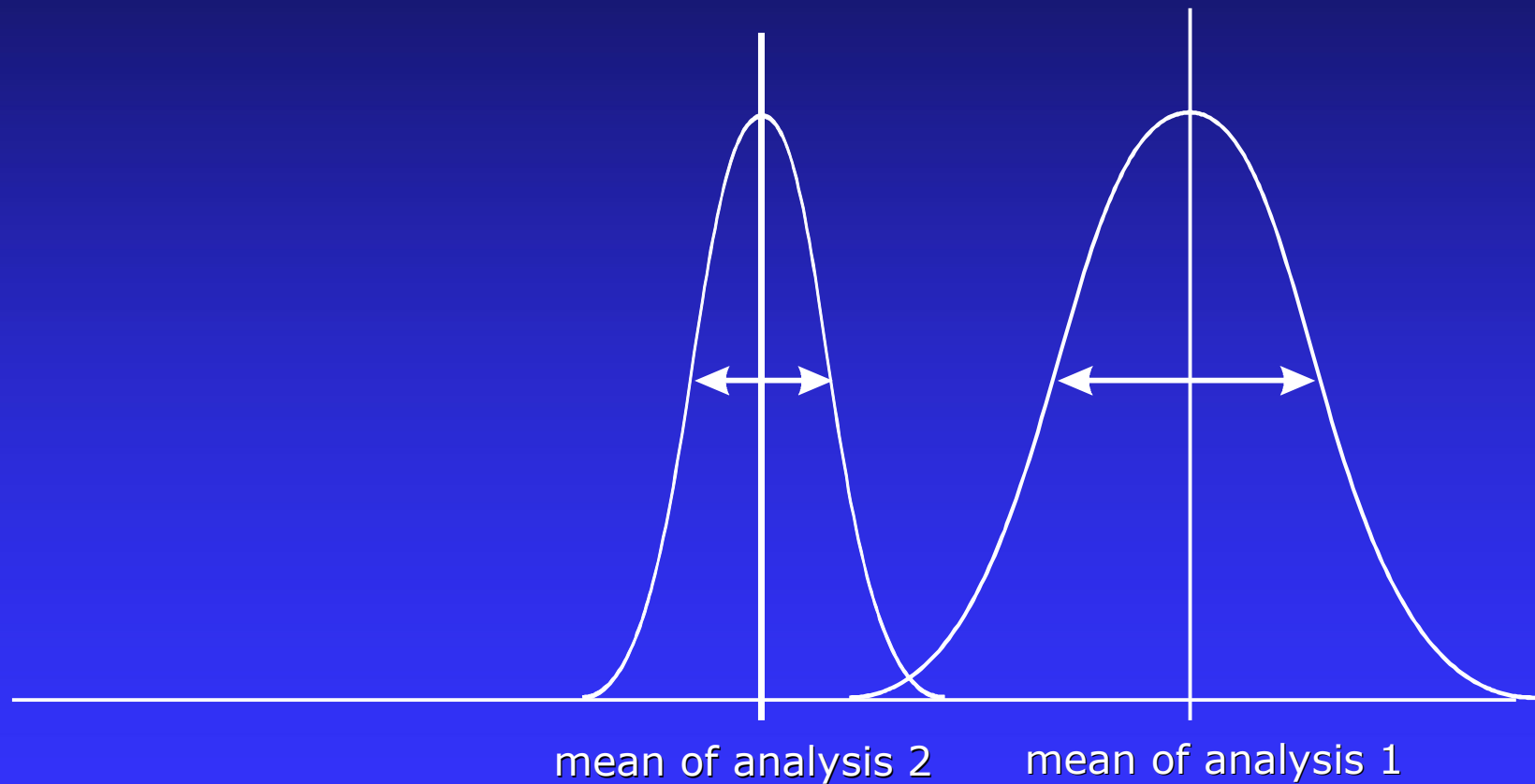
Random and Systematic Deviations as Error Type A and Type B



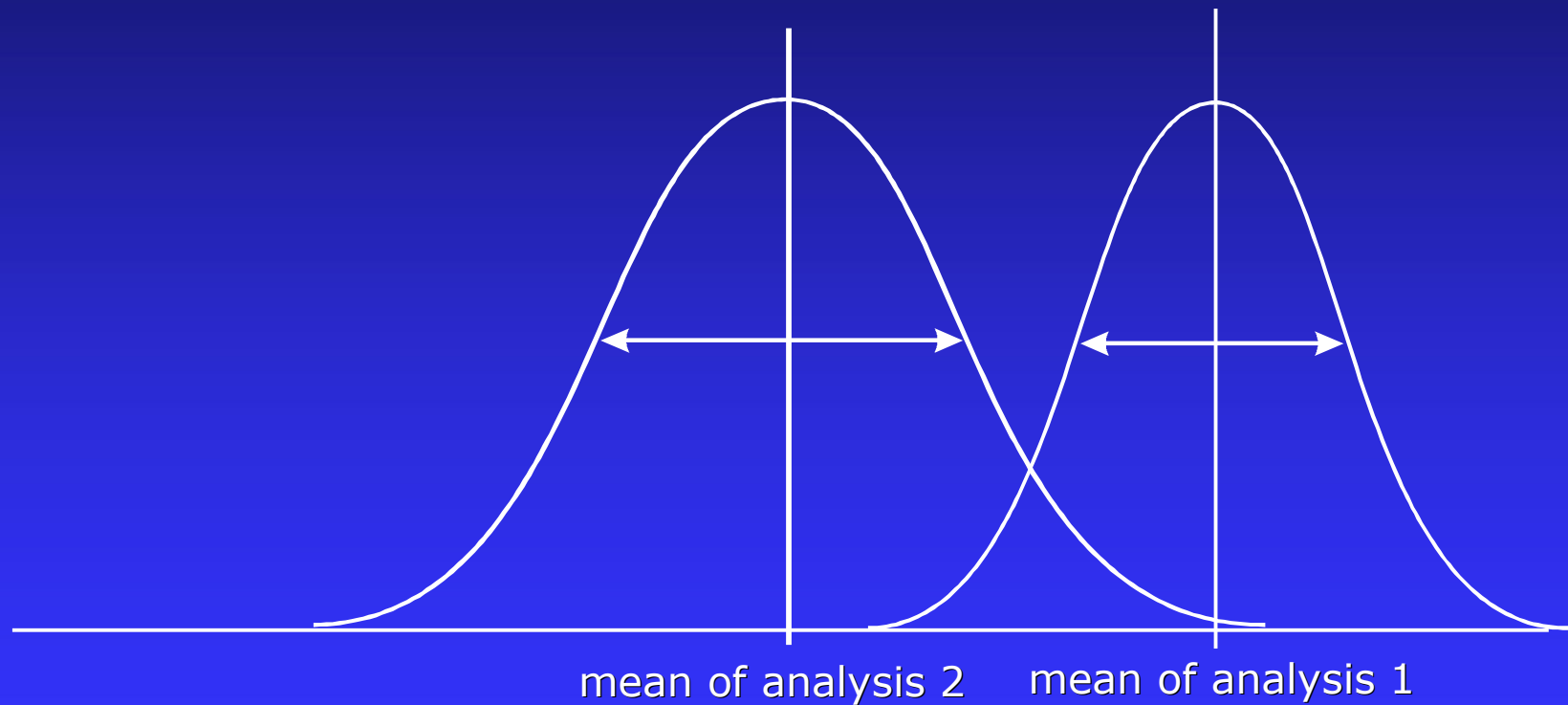
Random and Systematic Effects



Random and Systematic Effects



Random and Systematic Effects



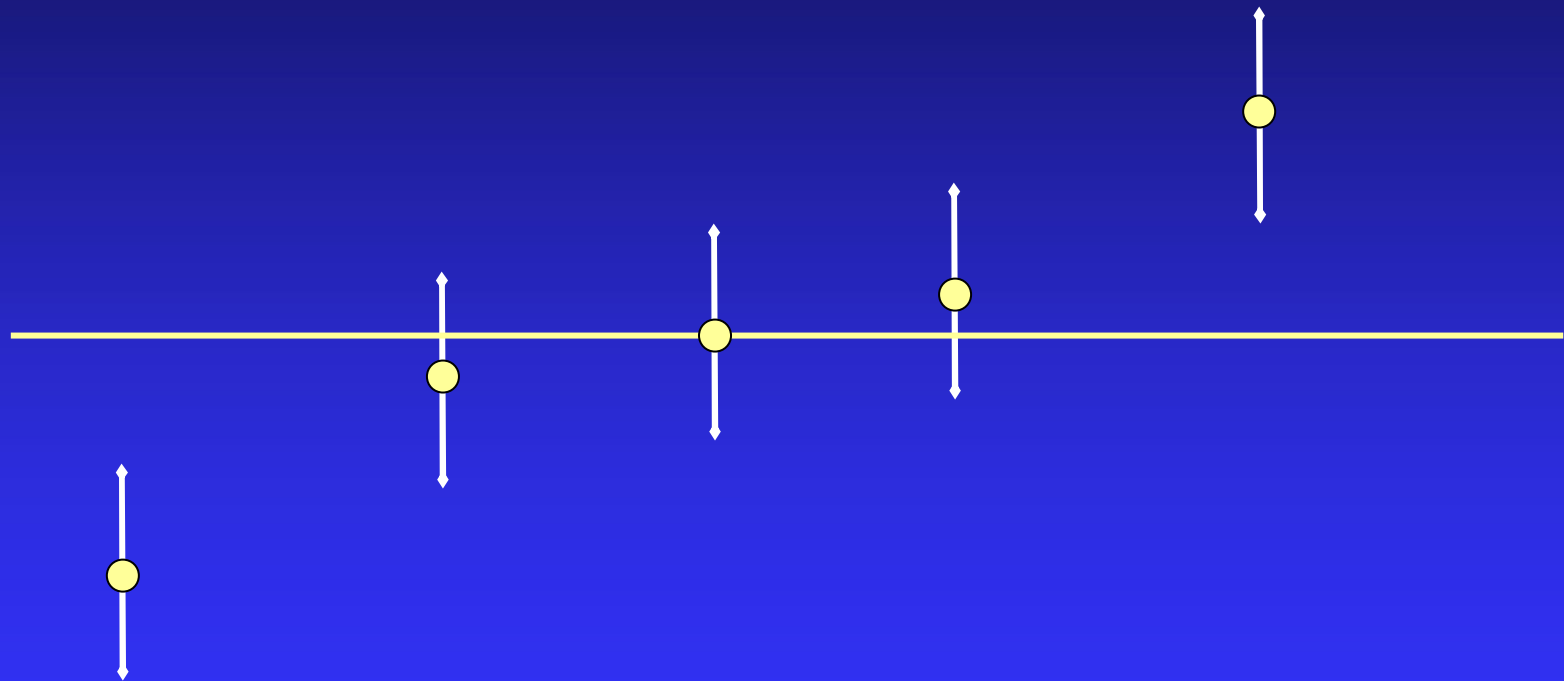
Major Goals of INC-1

- Comparability of Measurement Results
- Easy Reuse of Uncertainty Statements

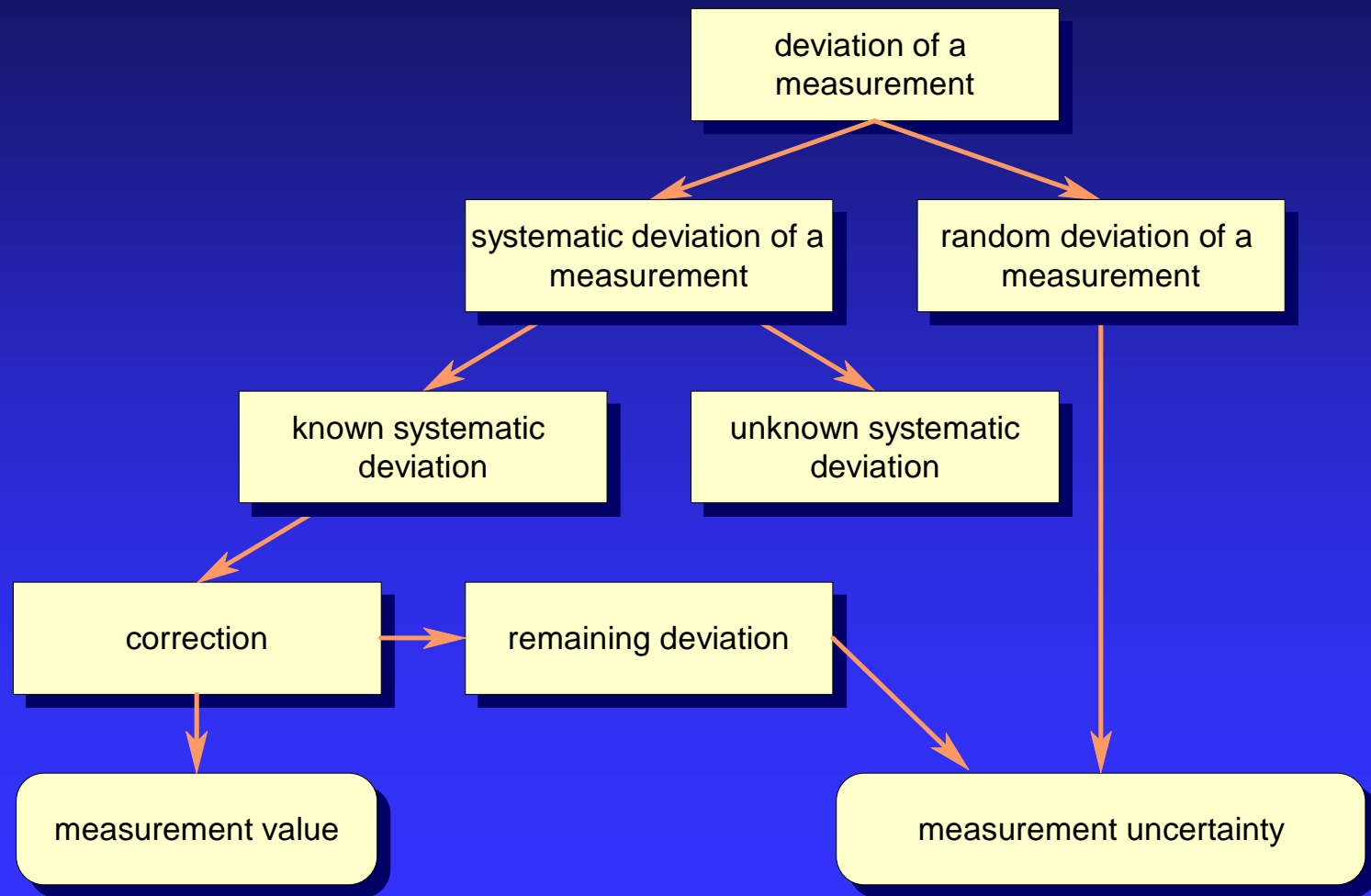
History

- 1977 Lack of international consensus on the expression of uncertainty in measurement (CIPM)
- 1980 Recommendation INC-1 Expression of Experimental Uncertainties (CIPM; 1981 & 1986)
- 1993 Guide to the Expression of Uncertainty in Measurement (ISO)
- 1995 Quantifying Uncertainty in Analytical Measurement (Eurachem/CITAC)
- 2000 Quantifying Uncertainty in Analytical Measurement Second Edition (Eurachem/CITAC Guide)

Comparing Results



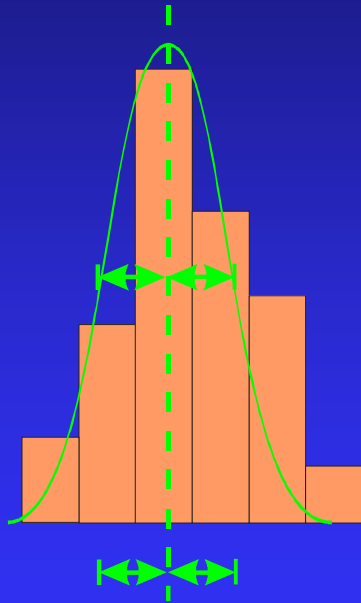
Major Goals of INC-1



Major Goals of INC-1

Concepts based on observable quantities

Uncorrected arithmetic mean of observations

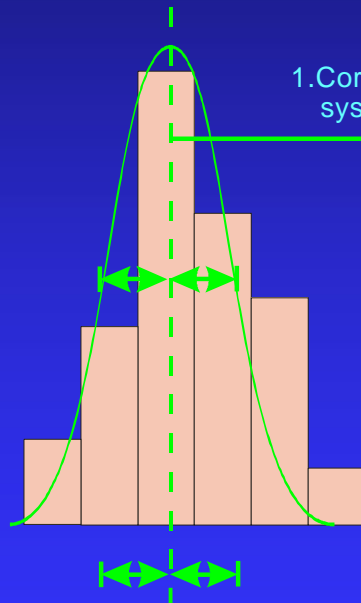


Standard uncertainty of the uncorrected mean due to the dispersion of the observations

Major Goals of INC-1

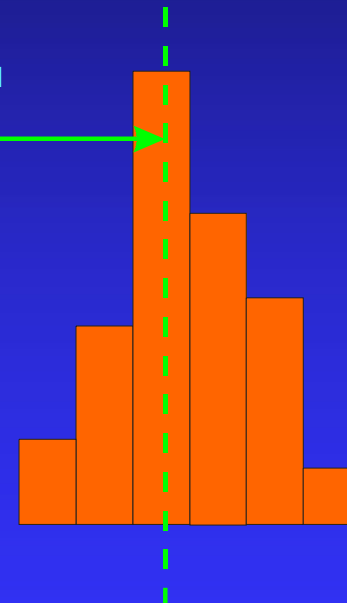
Concepts based on observable quantities

Uncorrected arithmetic mean of observations



Standard uncertainty of the uncorrected mean due to the dispersion of the observations

Corrected arithmetic mean of observations

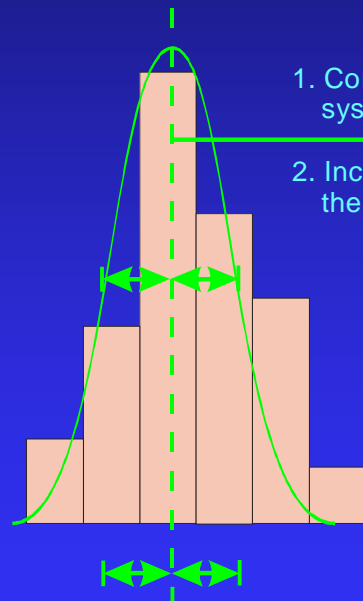


1. Correction for all recognized systematic effects

Major Goals of INC-1

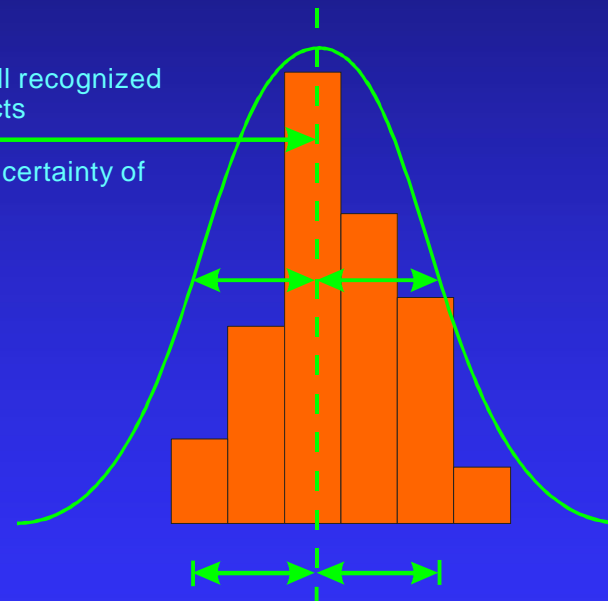
Concepts based on observable quantities

Uncorrected arithmetic mean of observations



Standard uncertainty of the uncorrected mean due to the dispersion of the observations

Corrected arithmetic mean of observations

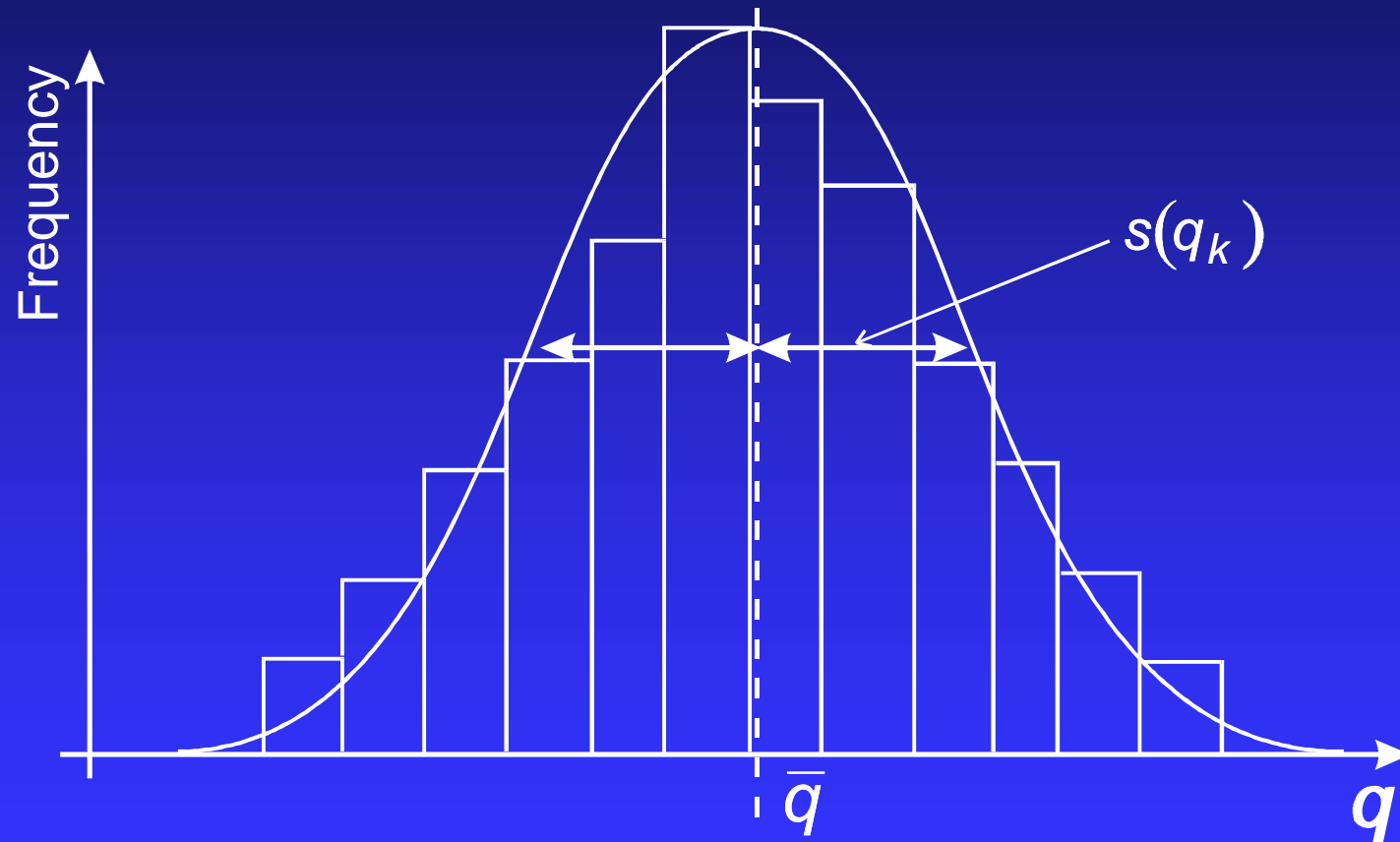


Combined standard uncertainty of the corrected mean. It comprises the uncertainty of the uncorrected mean due to the dispersion of the observations and the uncertainty of the applied correction

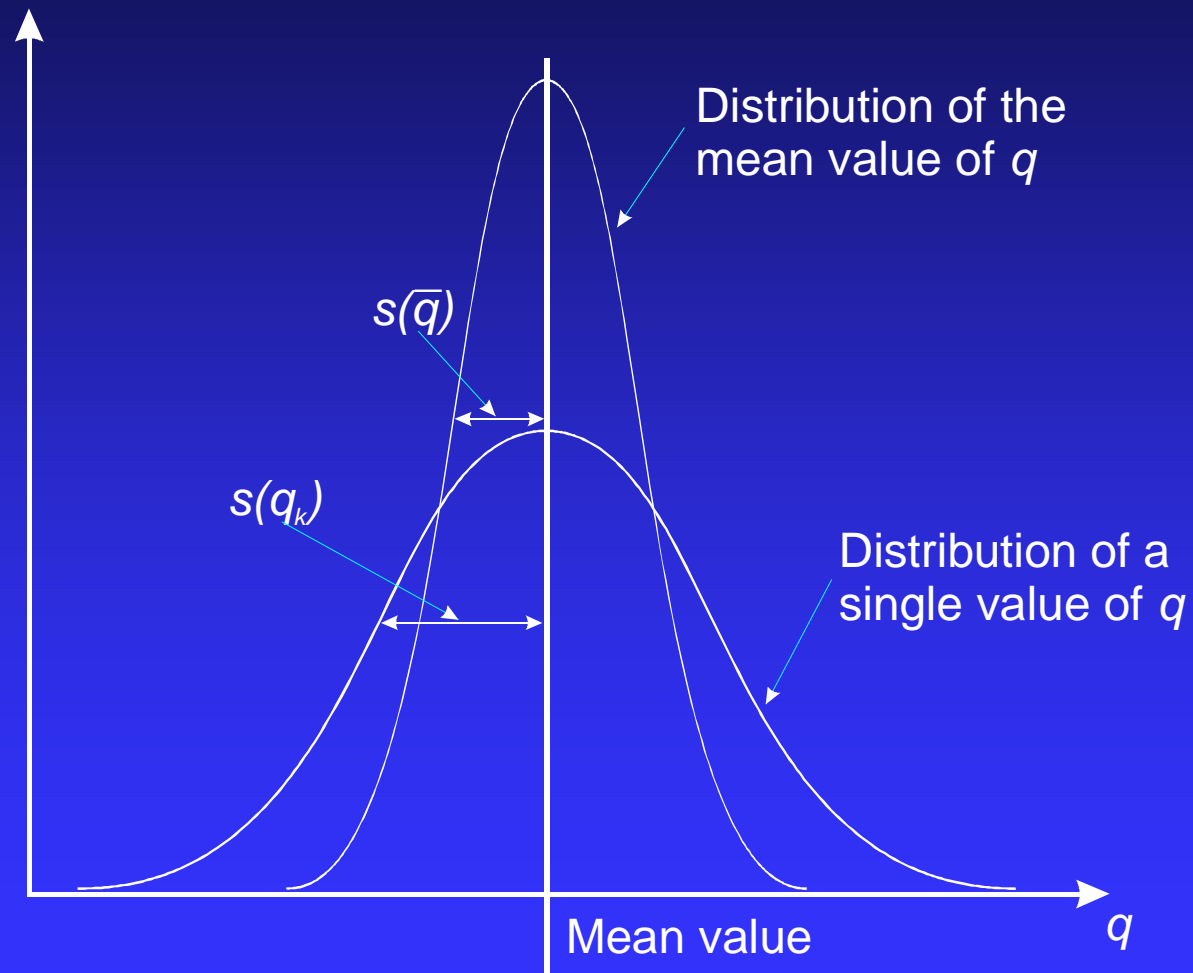
Definition of the Measurement Uncertainty

Parameter, associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand

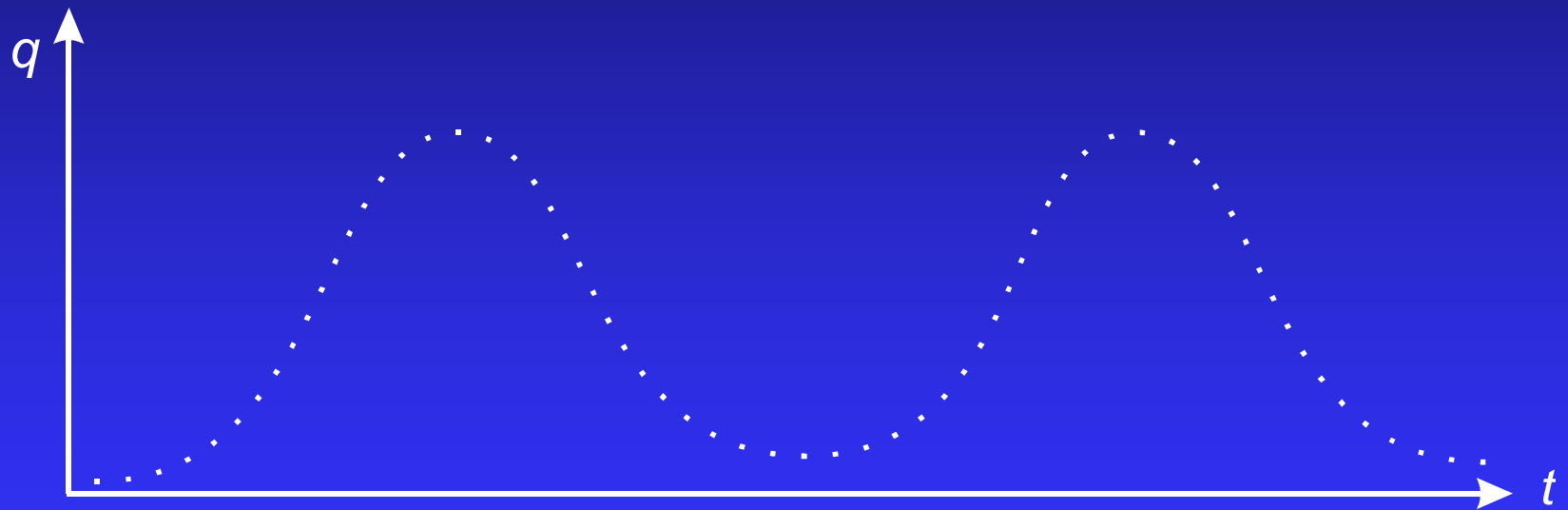
Statistical Evaluation of the Uncertainty



Statistical Evaluation of the Uncertainty



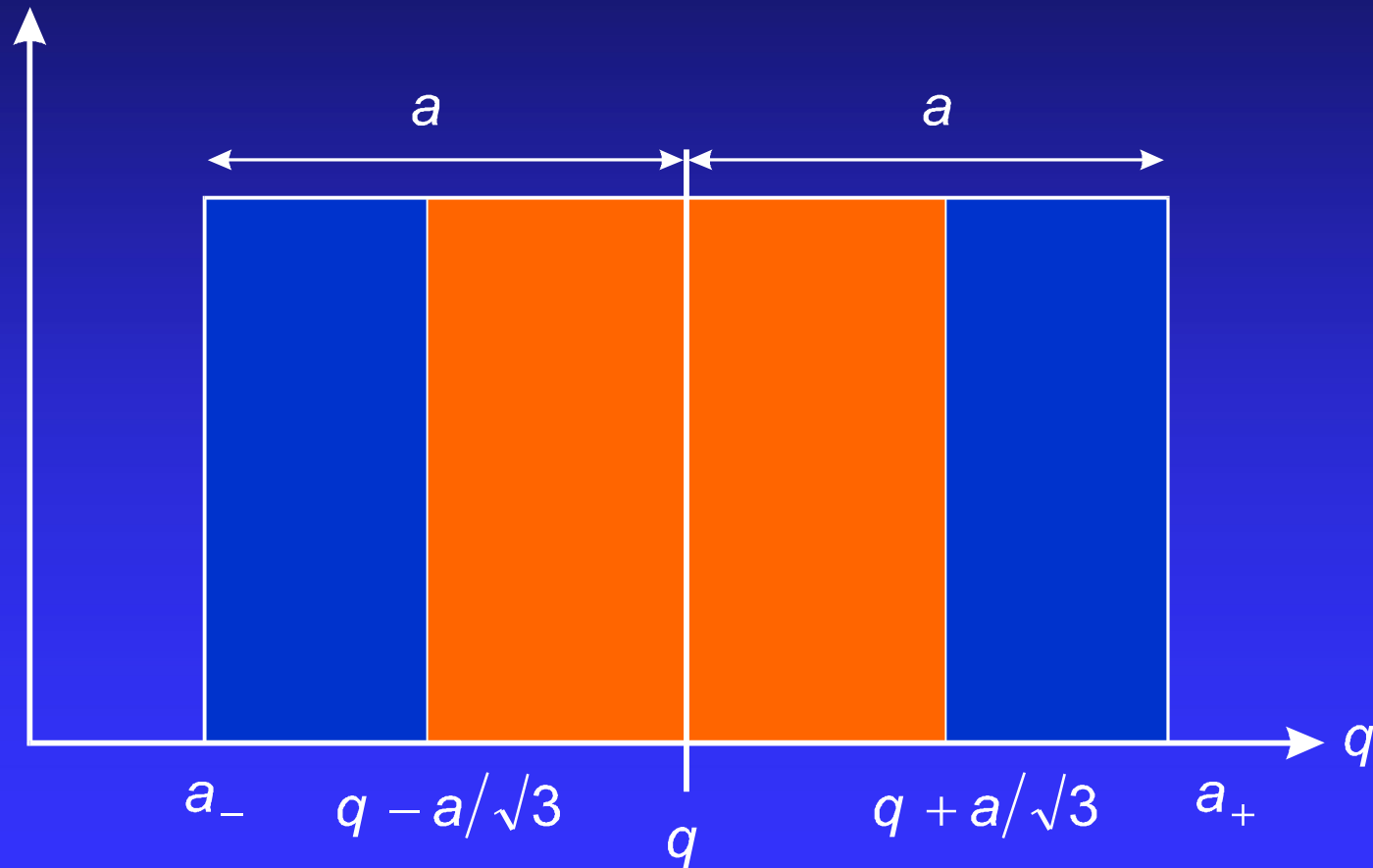
Statistical Evaluation of the Uncertainty



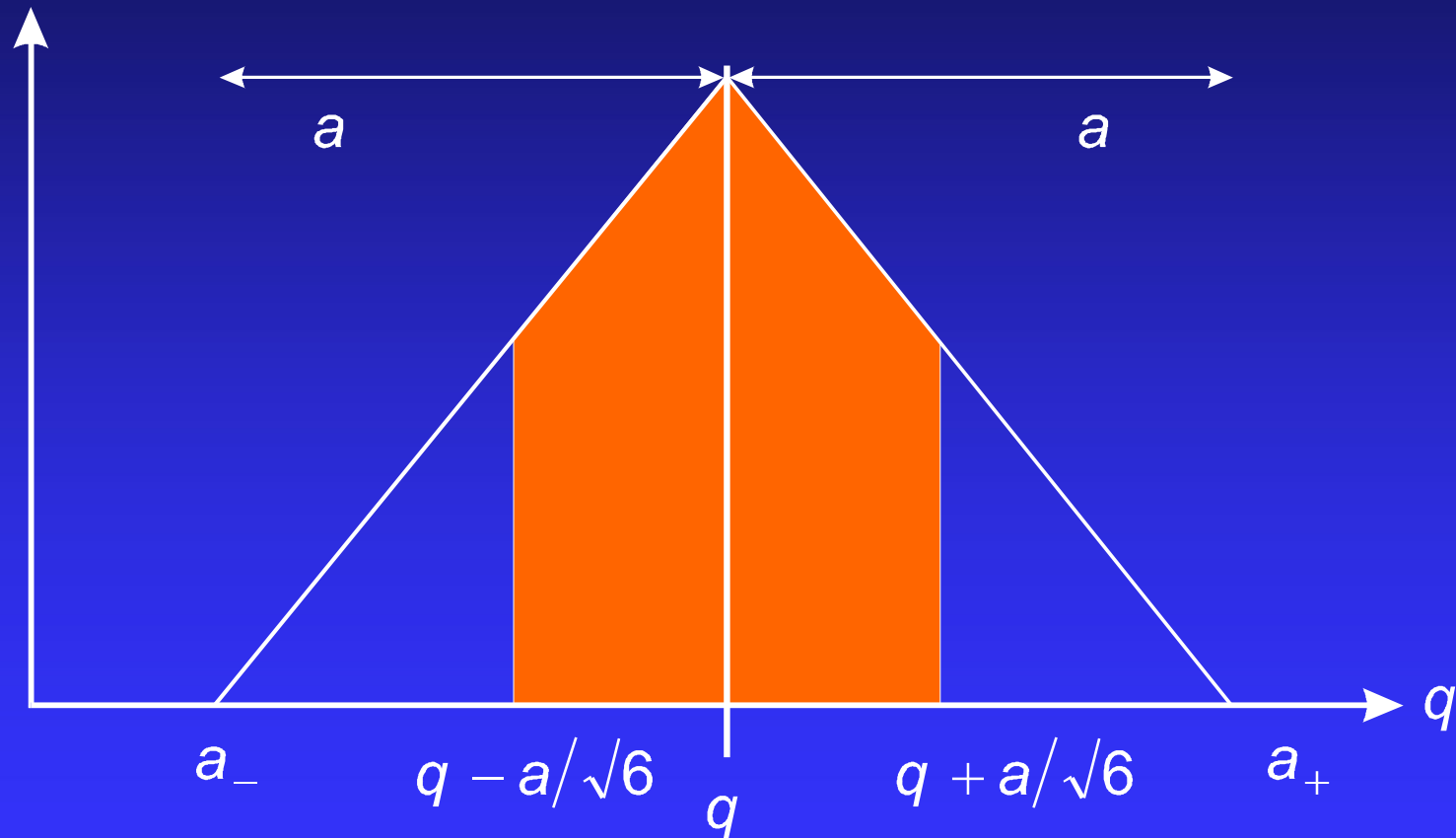
No Statistical Evaluation of the Uncertainty

content[ml]	tolerance[ml]
10	0.025
20	0.04
25	0.04
50	0.06
100	0.1
200	0.15
250	0.15
500	0.25
1000	0.4

No Statistical Evaluation of the Uncertainty



No Statistical Evaluation of the Uncertainty



Remarks on the Evaluation of Uncertainty

Number of measurements n	uncertainty of the uncertainty [%]
2	76
3	52
4	42
5	36
10	24
20	16
30	13
50	10

Calculating the Combined Standard Uncertainty

First order approximation (GUM)

$$\text{From } Y \approx f(\mu_1, \dots, \mu_n) + \sum_{i=1}^n \frac{\partial f}{\partial X_i} (X_i - \mu_i) + O((X_i - \mu_i)^2)$$

we obtain

$$E[Y] \approx f(\mu_1, \dots, \mu_n)$$

$$\text{Var}(Y) \approx \sum_{i=1}^n \left(\frac{\partial f}{\partial X_i} \right)^2 \text{Var}(X_i) + \sum_{j=1}^n \sum_{i=1}^n \frac{\partial f}{\partial X_i} \frac{\partial f}{\partial X_j} \text{Cov}(X_i, X_j)$$

Rules for Calculation

1.Rule: Addition and Subtraction

$$y = k(p + q - r + \dots)$$

$$u_c(y(p, q, r, \dots)) =$$
$$k \cdot \sqrt{u(p)^2 + u(q)^2 + u(r)^2 + \dots}$$

Rules for Calculation

2.Rule: Multiplication and Division

$$y = k(p \cdot q \cdot \dots)$$

$$u_c(y) =$$

$$y \cdot k \cdot \sqrt{\left(\frac{u(p)}{\bar{p}}\right)^2 + \left(\frac{u(q)}{\bar{q}}\right)^2 + \dots}$$